## Tracing Influence Across Interacting Networks


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Propagation processes in interconnected systems are rarely confined to a single network layer. For example, social interactions influence epidemiological exposure, and failures in one infrastructure network can cascade into others. Modeling these phenomena requires representations that capture not only the internal structure of each layer but also the directional and heterogeneous couplings among layers. However, widely used multilayer representations, most notably supra-matrices, collapse these interactions into a single aggregated operator, discarding alignment structure, blurring directional effects, and masking multiway relationships that govern cross-layer influence. Although tensor representations are a natural alternative, direct tensorization of adjacency matrices or time slices typically stacks layers as independent modes, without enforcing directional alignment or a shared latent geometry across layers, which obscures cross-layer propagation structure and offers limited interpretability beyond matrix-based formulations. To address these limitations, we introduce a tensorized embedding framework designed to preserve the geometry, directionality, and coupling mechanisms that shape cross-layer propagation in complex interconnected networks.

We study the problem of learning structured representations of propagation dynamics in interconnected networks to characterize how influence flows within and across layers. Our approach proceeds in three steps. (**Step 1**) We construct **directional embeddings** for each network layer via spectral decompositions that preserve orientation, flow asymmetry, and local structural organization. These embeddings define layer-specific latent spaces in which heterogeneous networks can be compared without suppressing intrinsic structural differences. (**Step 2**) We learn **alignment operators** that transform activity between embedding spaces, capturing how propagation effects transfer across layers. (**Step 3**) The resulting intra-layer embeddings and inter-layer alignments are organized into a **six-mode tensor representation**, which retains the native multiway structure of interconnected networks beyond matrix-based formulations. We apply **low-rank CP decomposition** to this tensor to identify dominant propagation components, enabling structured analysis of cross-layer influence patterns and their relative contributions. **Figure 1** summarizes this three-step pipeline.

Across synthetic benchmarks and co-simulated multilayer systems, our tensorized representation achieves **28 - 35% lower reconstruction error** and **19 - 26% higher classification accuracy** than supra-matrix and naive concatenation baselines while producing more stable propagation-mode signatures. Conceptually, it offers a principled perspective on multilayer diffusion by treating interconnected networks as genuinely multiway systems rather than flattened composites. This approach provides new analytical tools for understanding how dynamics propagate across interacting networks and opens opportunities for developing control strategies that leverage layer-specific and cross-layer structure.


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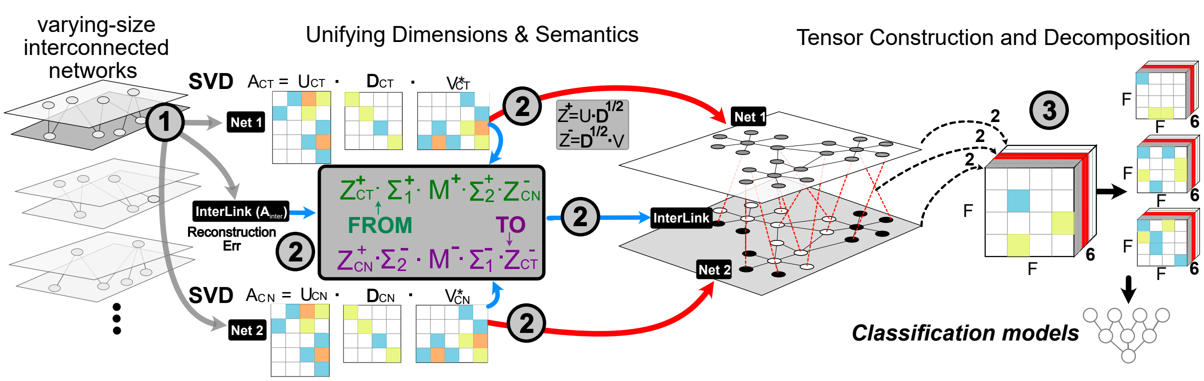


Figure 1: Alignment Learning for Interconnected Networks